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Two-dimensional Phase Unwrapping for Digital Holography

by Neal K. Bambha, Justin R. Bickford, and Karl K. Klett, Jr.

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14. ABSTRACT

This work describes a method for eliminating the mathematical ambiguity in the phase map for digital holographic images. The technique uses a pre-processing step to smooth the amplitude data and reduce noise in the phase data. The unwrapping algorithm uses both phase gradient calculations and an amplitude measure to determine the reliability of a given data point, and uses a non-continuous path following approach.

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Summary

We present a phase unwrapping algorithm for eliminating the mathematical ambiguity in the phase map for digital holographic images. The technique uses a pre-processing step to smooth the amplitude data and reduce noise in the phase data. The unwrapping algorithm uses both phase gradient calculations and an amplitude measure to determine the reliability of a given data point, and uses a non-continuous path-following approach. We demonstrated the utility of this algorithm by applying it to digital holographic data that we recorded. We demonstrated that the phase could be unwrapped correctly, and that the combination of amplitude and phase can be used to successfully reconstruct the object.

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1. Introduction

Digital holography is a means of producing three-dimensional (3-D) images using the digital image of the interference between a laser reference beam and a laser beam reflected from the object. We have described the basics of digital holography in ARL-TR-6189 (*1*). In this technique, both the amplitude and wrapped phase of the image is calculated via a Fourier transform. The phase is said to be wrapped because the values returned by the Fourier transform are limited to the range of $-\pi$ to π with discontinuities of 2π in the phase distribution. The true phase of the image must be calculated from the wrapped phase in a procedure called phase unwrapping or phase demodulation. For a two-dimensional image with noise and discontinuities, the unwrapping problem is difficult and has been researched for over a decade. It arises in other interferometric applications such as synthetic aperture radar. Many different types of algorithms have been described. These can be categorized into path dependent algorithms, where the order in which the pixels of the image are unwrapped is pre-determined, and path independent algorithms, which seek to minimize a global function. This report describes a path dependent algorithm that was determined to yield good results with the images in our experiment (*1*).

2. Methods, Assumptions, and Procedures

2.1 Algorithm

Path following algorithms generally use a reliability measure (also called quality measure or quality map) for each pixel in the image. Real images always contain noise, and the reliability value is a measure of the relative confidence in the wrapped phase value of the pixel given the noise. Pixels with the highest reliability are unwrapped first in order to minimize the propagation of errors (*2*–*6*). Filtering is often done before the unwrapping to reduce noise.

2.2 Reliability Function

We use a reliability function that uses both amplitude and phase information. A gradient (difference between a pixel and its neighbors) measure based on second differences is calculated from the phase values. Points with the lowest modulo 2π second differences with respect to their neighbors are determined to have higher reliability. The use of second differences provides a measure of the concavity/convexity of the phase function and reduces the effect of the carrier value compared to the modulation of the fringe pattern (*5*). The calculation of the second differences can be explained using figure 1.

(i-1, j-1)	(i, j-1)	(i+1, j-1)
(i-1, j)	(i, j)	(i+1, j)
(i-1, j+1)	(i, j+1)	(i+1, j+1)

Figure 1. Pixel organization for unwrapping algorithm.

The second difference D(i,j) of a pixel can be calculated by the equation as in reference 5, where φ represents phase, H represents horizontal differences, V represents vertical differences, and D_1 and D_2 represent diagonal differences:

$$D(i,j) = [H^2(i,j) + V^2\,(i,j) + D_1^2(i,j) + D_2^2(i,j)] \tag{1}$$

$$H(i,j) = [\varphi(i-1,j) - \varphi(i,j)] - [\varphi(i,j) - \varphi(i+1,j)] \tag{2}$$

$$V(i,j) = [\varphi(i,j-1) - \varphi(i,j)] - [\varphi(i,j) - \varphi(i,j+1)] \tag{3}$$

$$D_1(i,j) = [\varphi(i-1,j-1) - \varphi(i,j)] - [\varphi(i,j) - \varphi(i+1,j+1)] \tag{4}$$

The second differences are calculated for all pixels except those at the borders. Pixels with lower values of D are considered more reliable, since the true phase is assumed to be smooth. In addition, pixels whose intensity values are higher are also assumed to be more reliable since the signal-to-noise ratio is higher for these pixels. To combine both effects, we define the pixel reliability R as

$$R_p(i,j) = A(i,j)/D(i,j) \tag{5}$$

The use of both amplitude and phase differences in the reliability function is distinct from other works that only use the phase differences (*5*).

2.3 Unwrapping Path

Given the reliability of each pixel, we can define horizontal and vertical edges between all pixels, and define the reliability of an edge as the sum of the reliability of the pixel endpoints of the edge.

$$R_e = R_p\big(src(e)\big) + \ R_p(snk(e)) \tag{6}$$

where *src(e)* is the source pixel of edge *e* and *snk(e)* is the sink pixel of edge *e*.

The edges are then stored in an array and sorted by reliability. Edges with highest reliability are unwrapped first. As the pixels are unwrapped, they are placed into groups. Initially, all the pixels are not considered part of any group. When unwrapping the pixels on an edge, four possibilities exist:

1. Neither *src(e)* or *snk(e)* has been unwrapped yet. The pixels are unwrapped with respect to each other and placed in the same group.
2. One of the pixels *src(e)* or *snk(e)* belongs to a group while the other does not belong to a group. The pixel that does not belong to a group is unwrapped and placed in the other pixel's group.
3. Both pixels *src(e)* or *snk(e)* already belong to the same group. The edge can be removed from the array and the pixels do not need to be unwrapped.
4. Both pixels *src(e)* or *snk(e)* already belong to different groups. The two groups need to be unwrapped with respect to each other. Here, the group with the smaller number of pixels is unwrapped with respect to the larger group. The constant value that is required to wrap the pixel in the smaller group is added to the rest of the pixels in the smaller group, and the two groups are joined.

The process continues until all the pixels are part of a single group. It can be seen that the path followed is not spatially continuous, since it follows the reliability values of the edges.

3. Results and Discussion

We set up a digital holographic experiment as described in reference 1. We captured reference, object, and interferogram images at two different wavelengths and used the mathematical operations described in reference 1 to compute the phase and amplitude images used in this report. Figure 2 shows the phase images for both wavelengths λ_1 and λ_2. By themselves, the single wavelength phase images appear to have little relationship to the object. However, when we subtract the two images, interference fringes are evident. The phase difference image is equivalent to a single hologram recorded with a synthetic wavelength

$$\Lambda = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|} \tag{7}$$

Figure 2. Phase images for λ_1 (a) and λ_2 (b).

Figure 3 shows the phase difference image, zoomed to the area of the object. A 6x6 Gaussian weighted low-pass filter was applied to the data before processing. Here the interference fringes at the synthetic wavelength are clearly visible. This is the wrapped phase, which must be unwrapped to find the true phase of the object.

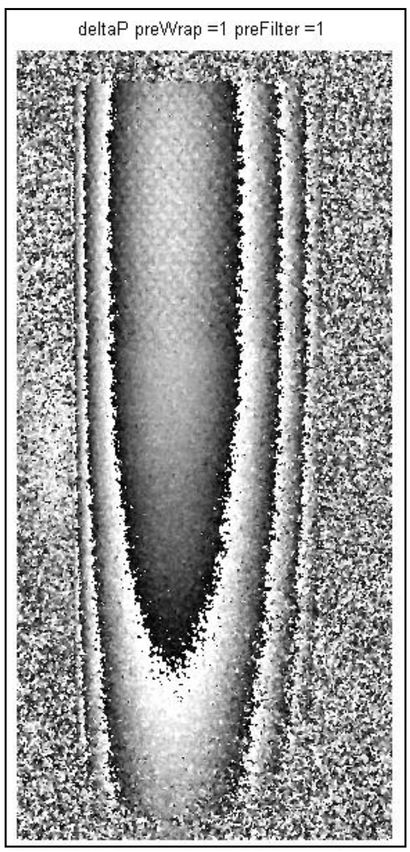

Figure 3. Phase difference image.

In order to see the noise reduction produced by the low-pass filter, figure 3 can be compared to figure 4, in which the low-pass filtering is not performed. Here, the fringes are not as distinct and more noise can be seen in the image.

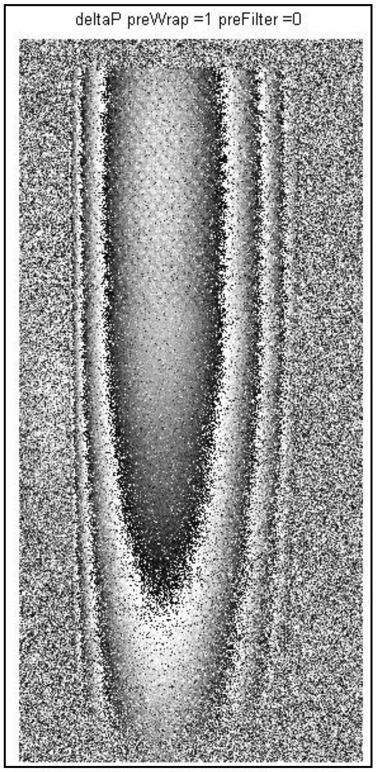

Figure 4. Phase difference image without low-pass filtering.

In figure 5, the amplitude image is shown. Both the real and virtual object can be seen.

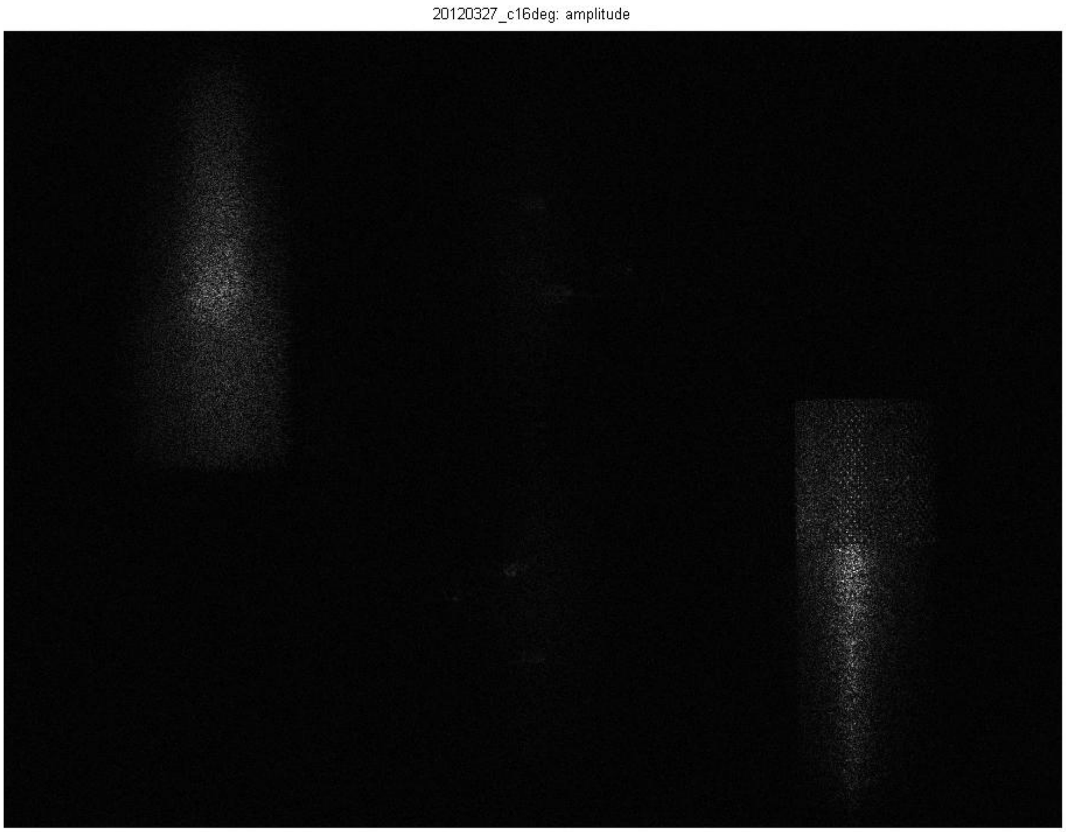

Figure 5. Amplitude image.

The image from figure 3 was input to the phase unwrapping algorithm, and the output from the algorithm is shown in figure 6. It can be seen that the phase unwrapping algorithm produced a smooth phase image. In figure 7, a 3-D view of the amplitude image overlaid on the phase image is shown. The phase is used to display the height of the object in the 3-D view. From this image, we can see that the phase was correctly unwrapped, and that the combination of amplitude and phase can be used to successfully reconstruct the object.

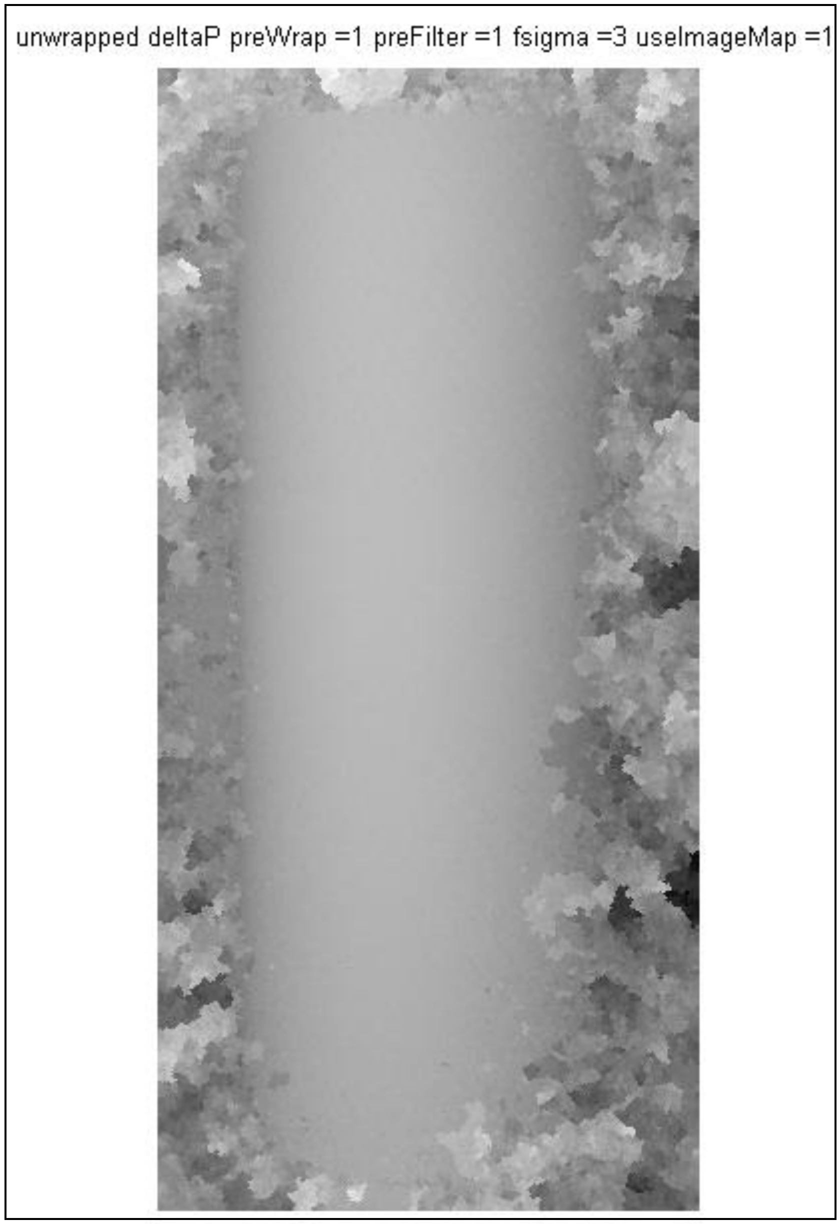

Figure 6. Unwrapped phase image.

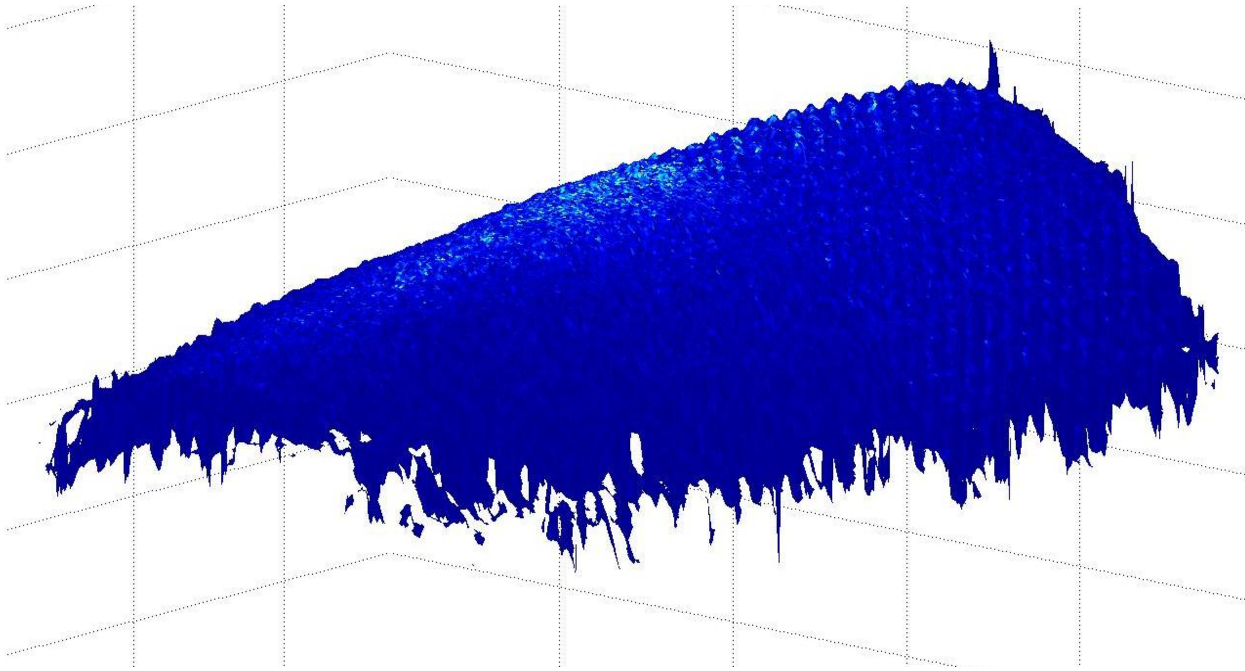

Figure 7. A 3-D view of the amplitude image overlaid on the phase image.

4. Conclusions

We have presented a phase unwrapping algorithm for eliminating the mathematical ambiguity in the phase map for digital holographic images. The technique uses a pre-processing step to smooth the amplitude data and reduce noise in the phase data. The unwrapping algorithm uses both phase gradient calculations and an amplitude measure to determine the reliability of a given data point, and uses a non-continuous path-following approach. We demonstrated the utility of this algorithm by applying it to digital holographic data that we recorded. We demonstrated that the phase could be unwrapped correctly, and that the combination of amplitude and phase can be used to successfully reconstruct the object.

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